A JAVA APPLET TO EXPLORE MULTIPLE REPRESENTATIONS OF FUNCTION IN A TEACHER EDUCATION COURSE

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Abstract

Schools in the Southern Regional Education Board-State Middle Grades Consortium promote that all students satisfactorily complete Algebra I or pass a pre-algebra test of proficiency and use algebra concepts to reason and solve problems. Therefore, it is vital that teacher education programs develop teachers who are prepared to meet this goal. JAVA applets created for use in a pre-service course may be used to explore multiple representations of functions in a problem solving context. The applet was developed to help students better understand mathematical modeling by exploring problems while using numerical, analytical, and graphical representations. Students are encouraged to work in collaborative pairs or triads and are required to write about mathematics. The applet is supplemented with recommended content standards and objectives, student performance descriptors and a grading rubric.

Background

In 2003, the Southern Education Regional Board, SREB, reported that in a recent *High Schools that Work* survey, 32 percent of career/technical students reported taking some form of Algebra I by the end of the eighth grade. Also, SREB reports that according to the National Assessment of Educational Progress Year 2000 Mathematics Assessment, about 25 percent of the nation's eighth-graders took Algebra I. (SREB,2000) Since schools in the SREB-State Middle Grades Consortium promote that all students satisfactorily complete Algebra I or pass a pre-algebra test of proficiency and use algebra concepts to reason and solve problems, it is vital that teacher education programs develop teachers who are prepared to meet this goal.

SREB noted that West Virginia has multiple ways for teacher candidates to achieve middle grade mathematics certification. Institutions in the State offer certification programs that overlap grade levels(2003). Usually when states have overlapping preparation programs, more attention is given to the secondary and elementary programs; the middle grades program is rarely a focus. At West Virginia University, undergraduates in a 5-year program may complete one of two programs of study to prepare for certification in middle school mathematics. One track is a 5-12 program in which the candidates earn a degree in mathematics, the other track allows students to attach an endorsement for middle school mathematics to a K-6 preparation program. Currently, all students in a K - 8 program at WVU are required to take pre-calculus (or college algebra and trigonometry) introductory statistics, a mathematics methods course and mathematics for K - 8 teachers. This program is currently under revision and issues such as student enrollment, faculty load, and budget constraints have caused those involved with the program development to delegate certain middle school mathematics objectives to the math course taught for K-8 teachers.

When choosing appropriate algebra objectives for the middle school certification program, faculty members referenced the National Council for Accreditation of Teacher Education, NCATE, Program Standards. For middle school mathematics teachers, NCATE assumes that the candidates have the equivalent of two years of algebra and a year of geometry in high school. A college program must be sufficient to allow successful completion of objectives in problem solving, reasoning, communication and connections. Specifically to mathematics content, candidates at accredited institutions must be prepared to use algebra to do such things as describe patterns, relations and functions, as well as to model and solve problems. Evidence must be given that teacher candidates learn to use appropriate technology to support the learning of mathematics and learn when and how to use student groupings such as collaborative groups, cooperative learning and peer teaching. (NCTM, 1998). Teacher candidates must also pass the PRAXIS test at the middle school mathematics level. Students must find such things as the domain and range, recognize properties of graphs, demonstrate an understanding of a physical situation and develop a model of it using such things as a graph, equation, or table. They should understand function notation and solve problems that involve linear and quadratic equations using a variety of methods. (Educational Testing Service, 2004)

Java Applet for Maximizing Area of Rectangular Gardens

Maximizing Area of Rectangular Gardens was developed for use in a course for pre-service teachers. In addition it may be used in an algebra course and addresses, in part, the high school algebra standard in West Virginia. The accompanying worksheets, which are not presented in this paper, may be used to guide the applet.

Content Standards are meant to be broad descriptions of what students should know and be able to do in a content area. They describe what students' knowledge and skills should be at the end of a sequence of study. The Maximizing Area applet was written taking into account standard 2 from the *West Virginia Content Standards and Objectives for Mathematics* (WVDE, 2004).

Standard 2: Students will:

- **demonstrate understanding of patterns, relations, and functions;**
- **represent and analyze mathematical situations and structures using algebraic symbols;**
- **uses mathematical models to represent and understand quantitative relationships; and**
- **analyze change in various contexts through communication, representation reasoning and proof, problem solving, and making connections within and beyond the field of mathematics.**

A content objective is an incremental step toward the accomplishment of content standards (WVDE, 2004). The java activity may be used to explore the following objectives from Algebra I, Algebra II, and PreCalculus. These are found in the *West Virginia Content Standards and Objectives* document. The format of each standard identification is **course_abbreviation.standard_number.objective_number**. So **A1.2.5** is for Algebra I and addresses standard 2, algebra, and is objective number 5.

A1.2.5 analyze a given set of data for the existence of a pattern numerically algebraically and graphically; determine the domain and range; and determine if the relation is a function.

A1.2.16 solve quadratic equations by graphing, factoring and quadratic formula.

A1.2.18 collect, organize, interpret data and predict outcomes using the mean, mode, median and range.

A2.2.8 solve equations containing radicals and **exponents**.

A2.2.9 define a function; find the domain, range, zeros; find the inverse of a function; **find the value of a function for a given element in its domain**; and perform basic operations on functions including composition of functions.

A2.2.10 explore families of functions: **recognize** linear, **quadratic**, absolute value, step, and exponential **functions; and convert among graphs, tables, and equations**.

A2.2.17 perform a quadratic regression and use the results to predict specific values of a variable. Identify the regression equation.

PC.2.1 investigate and sketch the graphs of polynomials and rational functions **using the characteristics of zeros**, upper and lower bounds, **y-intercepts, symmetry,** asymptotes and end behavior, **maximum and minimum points and domain and range.**

Performance Descriptors describe in narrative form how students demonstrate achievement of the content standards. In West Virginia, five performance levels have been adopted. The descriptors are meant to give teachers more information about the level of their students and are also used to explain student performance on statewide assessment instruments (WVDE, 2004). The following performance descriptors are suggested for the Maximizing Area applet.

 # Distinguished

The student demonstrates exceptional and exemplary performance with distinctive and sophisticated application of knowledge and skills that exceeds the standard in Algebra II. The student develops equations to solve practical application problems giving solutions in a clear, concise manner. The student finds domain, range and zeros of functions converting forms among graphs, tables and equations. The student solves quadratic equations over the set of complex numbers using various techniques confirming solutions both numerically and graphically in a clear concise manner and performs quadratic regressions using the regression equation to predict values.

 # Above Mastery

The student demonstrates competent and proficient performance and shows a thorough and effective application of knowledge and skills that exceeds the standard in Algebra II. The student finds domain, range and zeros of quadratic functions using graphs, tables and equations. The student solves quadratic equations over the set of complex numbers using various techniques, confirming solutions either numerically or graphically and performs quadratic regressions giving the regression equation.

 # Mastery

The student demonstrates fundamental course or grade level knowledge and skills by showing consistent and accurate academic performance that meets the standard in Algebra II. The student finds domain, range and zeros of basic quadratic functions using graphs, tables and equations. The student solves quadratic equations over the set of complex numbers confirming solutions numerically or graphically.

 # Partial Mastery

The student demonstrates basic but inconsistent performance of fundamental knowledge and skills characterized by errors and/or omissions in Algebra II. Performance needs further development. The student inconsistently finds the domain, range and zeros of simple quadratic functions and attempts to solve simple quadratic equations given a graph.

 # Novice

The student demonstrates substantial need for the development of fundamental knowledge and skills, characterized by fragmented and incomplete performance in Algebra II. Performance needs considerable development. The student graphs parabolas given a table and attempts to solve simple quadratic equations.

Suggestions for Grading the Worksheets

The author sets the total value of each example worksheet as 100 points. Some of the responses, usually the written ones, are weighted more heavily than others. In addition, up to 10 points of the 100 are awarded for the ability of students to work in a group and communicate mathematically. Worksheets turned in from only one student do not receive the 10 points assigned for mathematical communication.

It is suggested that a rubric be used to score the written responses in this applet. A rubric can be defined as a set of authoritative rules to give direction to the scoring of assessment tasks or activities (Key Curriculum, 1999). When holistic scoring is used, the item being evaluated is looked at as a whole. Points are not given for individual components. Also, when using this type of assessment, it is suggested that special qualitites in one aspect of a response may override weaknesses elsewhere.

Suggested Rubric for Written Responses

Responses receiving full credit should have the correct answer written in a clear detailed manner. These responses use sentences when required and contain few or no communication errors. There should be no question of the student(s)' understanding of the content surrounding the question being asked. All details should be addressed, including correct usage of the units of measurement being used, symbols required for communication and the identification of special types of numbers if required.

Responses receiving 3/4 credit include full correct answers except for a few minor arithmetic, symbolic, or communication errors. They still have clear and correct responses and explanations.

Responses receiving ½ credit include partial correct answers communicated with clear explanations. In some cases, the response may contain a full correct answer; however, it is not communicated in a clear manner. This usually means that a student puts down everything possible and wants the grader to pull out the correct parts. These responses may also be those with a correct solution, but contain no justification of the answer if specifically required.

Responses receiving 1/4 credit attempt to respond by beginning to develop a correct response. The response does not contain a correct answer; however, an attempt is made to communicate in a clear manner.

Responses receiving no credit are either blank, simply restate the problem, or show no real understanding of the question.

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